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A Computer Code for Gas-Liquid Two-Phase Vortex Motions: GLVM

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National Bureau of Standards
Center for Chemical Engineering
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July 1986

Prepared for

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John F. Kennedy Space Center
Kennedy Space Center, FL 32899

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☐ Document describes a computer program; SF-185, FIPS Software Summary, is attached.

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A computer program aimed at the phase separation between gas and liquid at zero gravity, induced by vortex motion, is developed. It utilizes an explicit solution method for a set of equations describing rotating gas-liquid flows. The vortex motion is established by a tangential fluid injection. A Lax-Wendroff two-step (McCormack's) numerical scheme is used. This program can be used to study the fluid dynamical behavior of the rotational two-phase fluids in a cylindrical tank. It provides a quick/easy sensitivity test on various parameters and thus provides the guidance for the design and use of actual physical systems for handling two-phase fluids.

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computer code; gas-liquid separation; numerical modeling; two-phase vortex motions

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Abstract

A computer program aimed at the phase separation between gas and liquid at zero gravity, induced by vortex motion, is developed. It utilizes an explicit solution method for a set of equations describing rotating gas-liquid flows. The vortex motion is established by a tangential fluid injection. A Lax-Wendroff two-step (McCormack's) numerical scheme is used. This program can be used to study the fluid dynamical behavior of the rotational two-phase fluids in a cylindrical tank. It provides a quick/easy sensitivity test on various parameters and thus provides the guidance for the design and use of actual physical systems for handling two-phase fluids.

Key Words: computer code; gas-liquid separation; numerical modeling; two-phase vortex motions

List of Notations

A_a, A_{ak}	Added mass coefficients
A_d, A_{dk}	Drag coefficients
$\bar{B}_k$	Body force density
C_{ij}, C_{pk}, C_{dk}	Generalized coefficients
d_1	Bubble diameter
d_2	Liquid (droplet) diameter
$\bar{M}_k$	Effective interfacial force density
n_a	Exponent used for w_{ak}
n_d	Exponent used for w_{dk}
p	Pressure
R	Tank radius
Re	$\rho_2 V_s R/\mu_2$, Reynolds number
R_{j1}, R_{j2}	Jet opening, $R_{j1} < r < R_{j2}$
R_1	Minimum radius considered in the numerical analysis
r	Radial coordinate
t	Time
V_j	Averaged jet velocity
V_{r1}	Gas radial velocity
V_{r2}	Liquid radial velocity
V_s	Velocity scale
$V_{\theta 1}$	Gas tangential velocity

$V_{\theta 2}$	Liquid tangential velocity
$\bar{V}_1$	Gas velocity
$\bar{V}_2$	Liquid velocity
w_{ak}	Weighting function for added mass coefficients
w_{dk}	Weighting function for drag coefficients
α_1	Gas volume fraction
α_2	$(1-\alpha_1)$, liquid volume fraction
γ	Exponent for diameter variation
θ	Circumferential coordinate
μ_1	Gas dynamic viscosity
μ_2	Liquid dynamic viscosity
μ_k^e	$\mu_k + \mu_k^t$ total effective viscosity
μ_k^t	Turbulence or eddy viscosity
ν_k	μ_k/ρ_k, kinematic viscosity
ρ_1	Gas density
ρ_2	Liquid density
ρ_k	Averaged density of k-phase
$\langle\rho^2\rangle$	$\alpha_1\alpha_2\rho_1\rho_2 + A_a(\alpha_1\rho_1 + \alpha_2\rho_2)$
ω	Angular velocity

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I. Introduction

Mechanical systems have been devised for producing artificial gravit fields to spin-up liquids in containers. These involve rotating mechani are cumbersome and, more importantly, have moving parts that can wear out. Here, liquid rotation created by fluid injection is considered. The detail analysis of the two-phase vortex model can be found elsewhere [1]. In this report, the details of the computer code are described.

The computer program was developed to study the fluid dynamical behavior of two-phase fluids in a tank at zero gravity. The phase separation between gas and liquid, induced by vortex motions, is of primary interest. The program utilizes an explicit solution method for a set of equations describing rotating gas-liquid flows. The vortex motion is established by a tangential fluid injection. A Lax-Wendroff two-step (McCormack's) numerical scheme is used in the computer program. This scheme uses a conservation form of a system of equations together with an auto time step feature.

The program was developed and tested on an HP-1000 minicomputer. The HP-1000's FORTRAN 77 is based on the American National Standards Institute (ANSI) 77 standard programming language FORTRAN (ANSI X3.9-1978). The HP FORTRAN 77 has extensions to provide a more structured approach to program development and more flexibility in computing for scientific applications. It fully implements the Military Standard Definition (MIL-STD-1753) of extensions to the ANSI 77 standard. In order to make the computer code more useful for other computer systems, modifications have been made so that the code is closer to the ANSI 77 standard and thus less system dependent. Some limited extensions are still kept in order to produce the code in the HP-1000. Since the graphic routines are system dependent and must be modified with their equivalents at each computing facility, the original graphic code has not been included in this report. All

lines preceded by "*V" are originally adopted to use the vector operation package supplied by Hewlett Packard. The speed of the code can be increased by replacing many "do-loop" operations in the code with high speed vector operations. Little effort is required to incorporate the vector operation into the code if the vector operation package is in the system.

Thus with limited effort, the program can be adapted easily to most systems accepting the ANSI 77 standard FORTRAN. For example, the EMA (Extended Memory Area) statements may have to be removed from the code for some computers. Also double precision real numbers (Real * 8) could be replaced by single precision real numbers.

II. Model Equations

The vortex induced model is based on a two-phase, two-fluid continuum [2]. It incorporates several interactions between phases; namely fluid drag and virtual mass effects and it can be modified to include additional interaction effects. Detailed analysis of the model has been reported in Ref. 1. A brief summary of the system of equations is given below.

The equations for the conservation of mass and momentum for the two fluid two-phase model in an one-dimensional, axisymmetric case (i.e. $\frac{\partial}{\partial z} = \frac{\partial}{\partial \theta} = 0$) are:

$$\alpha_1 + \alpha_2 = 1$$

$$\frac{\partial r\alpha_k}{\partial t} + \frac{\partial r\alpha_k V_{rk}}{\partial r} = 0$$

$$\frac{\partial r\alpha_k V_{rk}}{\partial t} + \frac{\partial r\alpha_k V_{rk}^2}{\partial r} - \alpha_k V_{\theta k}^2 = -\alpha_k C_{pk}\, r\, \frac{\partial p}{\partial r}$$

$$+ \alpha_k \sum_{\ell=1}^{2} C_{k\ell} \left(\frac{\partial r\alpha_\ell \tau_{rr\ell}}{\partial r} - \alpha_\ell \tau_{\theta\theta\ell} + \alpha_\ell \rho_\ell\, B_{r\ell} r\right)$$

$$+ \alpha_k\, C_{dk}\, r\, (V_{r1} - V_{r2})$$

$$\frac{\partial r\alpha_k V_{\theta k}}{\partial t} + \frac{\partial r\alpha_k V_{rk} V_{\theta k}}{\partial r} + \alpha_k V_{rk} V_{\theta k} = \alpha_k \sum_{\ell=1}^{2} C_{k\ell} \left(\frac{\partial r\alpha_\ell \tau_{r\theta\ell}}{\partial r} + \alpha_\ell \tau_{r\theta\ell} + \alpha_\ell \rho_\ell\, B_{\theta\ell}\, r\right)$$

$$+ \alpha_k\, C_{dk}\, r\, (V_{\theta 1} - V_{\theta 2})$$

for $k = 1$ and 2 and with

$$C_{p1} = (\alpha_1 \alpha_2 \rho_2 + A_a)/\langle\rho^2\rangle$$

$$C_{p2} = (\alpha_1 \alpha_2 \rho_1 + A_a)/\langle\rho^2\rangle$$

$$C_{11} = (\alpha_2 \rho_2 + A_a)/\langle\rho^2\rangle$$

$$C_{12} = C_{21} = A_a/\langle\rho^2\rangle$$

$$C_{22} = (\alpha_1 \rho_1 + A_a)/\langle\rho^2\rangle$$

$$C_{d1} = -\alpha_2 \rho_2 A_d / \langle \rho^2 \rangle$$

$$C_{d2} = \alpha_1 \rho_1 A_d / \langle \rho^2 \rangle$$

and

$$\langle \rho^2 \rangle = \alpha_1 \alpha_2 \rho_1 \rho_2 + A_a (\alpha_1 \rho_1 + \alpha_2 \rho_2)$$

The effective stresses are modeled as

$$\tau_{rrk} = 2\mu_k^e \, \partial V_{rk}/\partial r$$

$$\tau_{r\theta k} = \tau_{\theta rk} = \mu_k^e \, r \, \partial(V_{\theta k}/r)/\partial r$$

$$\tau_{\theta\theta k} = 2 \, \mu_k^e \, V_{rk}/r$$

with

$$\mu_k^e = \mu_k + \mu_k^t$$

and the interfacial forces are modeled in the form of

$$\overline{M}_1 = A_d(\overline{V}_2 - \overline{V}_1) + A_a \frac{d}{dt} (\overline{V}_2 - \overline{V}_1).$$

$\overline{M}_1$ is the force density acting on the phase 1 by the phase 2. A_a and A_d are the added mass and drag coefficients, respectively.

The incompressibility condition is reduced to $\alpha_1 V_{r1} + \alpha_2 V_{r2} = Q_r/r$, where Q_r is the net radial outflow.

In the program $Q_r = 0$ is assumed, since the mixture pumped out is injected immediately back into the tank at the nearby location. The net volume or mass in the system is effectively unchanged except for the net change on the angular momentum. Thus, the pump system (withdrawal and injection) acts as a body force

on the mixture at the nozzle location. The net momentum gain is thus the momentum introduced into the system minus the local momentum pumped out. Therefore, we will model this pumping dynamic by body forces without considering the mass transfer. That is, the body force density $\alpha_k \rho_k \overline{B}_k r$ will be replaced by the net momentum gain, $\frac{\alpha_k \rho_k V_j}{2\pi} (V_j \overline{n} - \overline{V})$ at the nozzle location, where V_j is the injection speed.

III. Numerical Method

The complete solution of the complicated system of equations can only be obtained through numerical methods. An improved Lax-Wendroff, two-step scheme, (also referred to as MacCormack's method) [3, 4] is adopted for solving this time-dependent problem. This non-centered differencing scheme, using a full step backward prediction and forward correction version, requires no explicit artificial viscosity if a proper stability condition is satisfied. Using this technique for solving fluid flow problems is very efficient and has been in widespread and successful use for some time. It is good both for the time-accurate computation of steady and unsteady flow problems. The general features of the scheme are: i) its explicitly conservative form, ii) it is a two-step predictor-correction type, iii) it is three point, two level - that is, the solution of f_i^{n+1} at level n+1 depends only on three values of f_i^n at level n, and iv) it is second-order accurate in time and in space.

For using the MacCormack's numerical technique, the system of equations can be expressed in the conservative form as:

$$W_t = F_r + P_r + gG_r + S$$

Here the subscripts (t and r) denote partial differentiation with respect to t and r, respectively, and W, F, P_r, gG_r and S are column matrices with five

elements. All the components of F, P_r, gG_r and S can be regarded as functions of the components of W which are the independent variables. The fundamental theory of the MacCormack's scheme is briefly given below.

For second order accuracy, the solution could be written as

$$W^1 = W^o + \Delta t\, W_t{}^o + \frac{(\Delta t)^2}{2} W_{tt}{}^o$$

$$= W^o + \frac{\Delta t}{2} W_t{}^o + \frac{\Delta t}{2} (W_t{}^o + \Delta t\, W_{tt}{}^o)$$

$$= \frac{1}{2} (W^o + \Delta t\, W_t{}^o) + \frac{1}{2} (W^o + \Delta t\, W_t{}^p)$$

$$= \frac{1}{2} (W^p + W^c)$$

where

$W^p = W^o + \Delta t\, W_t{}^o$ is the predicted value,

and

$W^c = W^o + \Delta t\, W_t{}^p$ is the corrected value.

The superscripts denote the time-level of the information and subscripts denote the partial derivative with respect to either time t or space r. Specifically, superscripts 0 and 1 are the initial and the completely advanced time (here two steps) plane; p and c are the predicted (1st step) and corrected (2nd step) time plane. Thus, $W_t{}^o$ is the time derivative of W evaluated at the initial time, and $W_t{}^p$ is time derivative of W evaluated at the predicted time.

Fig. 1 shows the diagram of the two step difference scheme used in the computer program. Due to the difference scheme, the spatial location after each step in time is a half grid off from the original one. Thus, the spatial offset

which resulted from a backward predicting step will cancel with those of the forward correcting step.

Numerically, the predicted values are

$$W_i^p = \frac{1}{2}\,(W^p_{i-1/2} + W^p_{i+1/2})$$

where

$$W_{i-1/2}^{\,p} = \frac{1}{2}\,(W_{i-1}^{\,o} + W_i^{\,o}) + \Delta t\; W_t^{\,o}$$

$$= \frac{1}{2}\,(W_{i-1}^{\,o} + W_i^{\,o}) + \frac{\Delta t}{\Delta r}\,[(F_i^{\,o} - F_{i-1}^{\,o}) + \frac{(g_i^{\,o} + g_{i-1}^{\,o})}{2}\,(G_i^{\,o} - G_{i-1}^{\,o})]$$

$$+ \frac{\Delta t}{2}\,(S_i^o + S_{i-1}^{\,o}) + \Delta t\;\hat{P}^p_{i-1/2}\,.$$

and the corrected value is evaluated at the predicted time place, that is at $W_{i+1/2}^{\,p}$. Thus

$$W_i^c = W_i^{\,o} + \Delta t\; W_t^{\,p}$$

$$= W_i^{\,o} + \frac{\Delta t}{\Delta r}\,[(F_{i+1/2}^{\,p} - F_{i-1/2}^{\,p}) + \frac{(g_{i+1/2}^{\,p} + g_{i-1/2}^{\,p})}{2}\,(G_{i+1/2}^{\,p} - G_{i-1/2}^{\,p})]$$

$$+ \frac{\Delta t}{2}\,(S_{i+1/2}^{\,p} + S_{i-1/2}^{\,p}) + \Delta t\;\hat{P}_i^{\,c}$$

Here $\hat{P}_{i-1/2}^{\,p}$ and $\hat{P}_i^{\,c}$ are the pressure correction terms at half and full time steps respectively. Thus, for each time step, the advance is carried out in two

steps: a full step backward predictor, and then a forward corrector. As indicated in the diagram, the subscript i is the regular mesh spatial location at which solution is to be advanced, $i \pm 1$ is the spatial location of regular mesh points immediately to the right and left of the location i, $i \pm 1/2$ is the location midway between i and i + 1 or between i - 1 and i at the predictor plane. Thus, for each time step as the procedure advanced, the outermost data points at the boundary are not updated through the numerical scheme. The values at the boundary are to be given through some suitable boundary conditions. The numerical procedure utilizes a uniformly preselected spatial mesh and variable time increment. To avoid a singularity at the center of the core region, a finite radius R_i is used for the inner boundary. The tank radius R is the outer boundary. The time step is determined at each time step to ensure numerical stability [5]. For a finite grid size Δr, the maximum time step Δt is given by

$$\Delta t_k = 1/[|C_{dk}| + |V_{rk}|/\Delta r + \frac{2}{\Delta r^2} (\alpha_1 \mu_1^e C_{k1} + \alpha_2 \mu_2^e C_{k2})]$$

where k = 1 and 2. The minimum Δt_k (with some rounding off) is used for the time step. Normally, the technique with the time step condition gives fairly good numerical stability. However, in critical conditions numerical damping can be added either for damping oscillations due to large gradients or for accelerating the calculation by increasing the time step. A damping factor, D thus was added in the program as

$$W_i^{1D} = W_i^{1} (1-D) + (W_{i-1}^1 + W_{i+1}^1 - W_i^1) D$$

where W_i^1 is the value obtained based on the two-step scheme, and W^{1D} is the value after the damping factor D is added. A typical value of D = 0.2 can be used for debugging the program. If no damping factor is desired, D = 0 should be used.

The computer program was written in a Fortran 77 based computer code. The code will permit evaluation of the effects of various parameters which control the fluid dynamical behavior. These include tank size, fluid properties, such as density and viscosity, etc., characteristic gas bubble and liquid drops sizes, and relative location of injection nozzles.

A sample input and its output are shown on Exhibits A and B, respectively. The initial conditions for the gas and liquid volume fractions are taken to be 25% gas and 75% liquid. These fractions are uniformly distributed over the circular cross-section of the cylindrical tank. Initially both fluids are at rest. Other parameters can be found in the sample input in Exhibit A. The resulting velocity distributions and gas volume fraction as function of time for the sample run are given in Figs. (2) and (3) respectively. The velocity distributions are displayed along equally spaced rays at different times to enable clear observation. These velocity vector fields indicate all flows are primarily in angular rotation with gas phase tending to move inward and liquid phase trying to move outward, as expected. As the result of these radial movements, the volume fraction distribution is also changed with time. And as expected, the gas volume fraction is increasing at the inner region and decreasing at the outer region as shown in Figure 3. More detailed results have been reported in Ref. 1.

IV. Program Details

The complete computer code is listed in Appendix A. The code consists of a main program, GLVM and several subroutines. It is written in subroutine form such that each subroutine performs an individual task. Each logical part is clearly isolated and it can be easily modified to reflect different modelings for the interfacial forces. The interactive input mode with self-instruction is used for easy parameter insertion. Many instructive internal documentations are included in the program. In the following, each subroutine is listed with a brief description of its major function.

1) GLVM, the Main Program.

*To initialize data and start the program: Logical unit to save data (LUS), (Logical Unit is 1 for terminal, and 6 for printer), job identification notes (NOTES), data file name for saving data (NAMR), initial time (TO), final time (TMAX), time interval for data output (DTPRT), etc.

*To control the calling sequences to the other subroutines.

*To check the time step.

*To save, print (and plot) the output data.

*To obtain the predicted and corrected values in the two step, numerical scheme.

*To impose boundary condition.

*To update the data, time, and step number for the new time step.

*To provide a shutdown procedure either in normal (e.g., $t > t_{max}$) or abnormal (e.g., Δt is too small) conditions.

2) INIT, Initialization.

*To input the test parameters, initial conditions and set-up the initial column matrix W.

Default values are provided for most of the parameters. The default values are listed at each interactive input step. If the default value is acceptable, a comma "," is inputted.

Some of the relevant symbols used are listed below:

ALMT	Limit values of α_1, ALMT(1) < α_1 < ALMT(2).
DAMP	Numerical damping factor. Normally set to 0.
DEN1D2	Density ratio, ρ_1/ρ_2
DO	Base diameter, i.e., $d_k = d_{ok}\alpha_k^{\gamma}$
DS	Density scale = ρ_2
EVF	μ^t/μ effective eddy viscosity factor.
GAMMA	Diameter exponent γ
IVTX	Type of simple initial flow: 0 = at rest, 1 = simple rotation, 2 = Hammel-Oseen Vortex, 3 = G.I. Taylor Vortex. This is needed only when there is no data file (NAMR) given for an initial condition.
IW	Boundary condition at wall (for tangential component). 0 = free-slip (no-skin friction), 1 = non-slip. Also, when I_W = 1, a factor of $(1-r)^{0.1}$ was included on IVTX flow to simulate an initial power law boundary layer.
MM	Size of data arrays, MM $\geq$ NG. MM appears in many subroutines.
MU	μ_k, dynamic viscosity for phase k.
MUEF	(1 + EVF) * MU. Effective viscosity.
MU1D2	μ_1/μ_2, Viscosity ratio.
NA, ND	n_a, n_b, Exponents for weighting function for drag and added mass coefficients, A_a and A_d.

NAMR Data file name for initial condition, if any.

NG Number of grid points used.

OMEGA ω, initial rotation speed, $V_\theta = \omega r$, when IVTX > 0.

PS Pressure scale, DS * VS^2

QJ,VJ Injection flow rate and speed.

RE Reynolds number, $\rho_2 V_s R/\mu_2$

RJ1,RJ2 Jet opening, RJ1 < r < RJ2

RPEAK Location of the peak speed of the initial vortex, if IVTX > 1

RTANK Tank radius = Length scale LS.

VPEAK Peak speed of the initial vortex, if IVTX > 1.

TJ1,TJ2 Injection time, TJ1 < t < TJ2.

TS Time scale = LS/VS

VS Velocity scale.

The format of the data file for the initial condition (if any) is a six column and NG row data file, where NG is the number of grid points. The column sequence is K, $\alpha_1(K)$, $V_{r1}(K)$, $V_{r2}(K)$, $V_{\theta 1}(K)$, $V_{\theta 2}(K)$, where K is the grid point number, and the rest of the terms are the gas volume fraction, radial gas velocity, radial liquid velocity, tangential gas velocity and tangential liquid velocity respectively. The data format is free.

3) DELA, ΔA

To determine the fraction of grid size in which the injection is made. $0 \le \Delta A \le 1$. This is used to define the location of jet. The region of injection could cover several full or fractions of grid sizes.

4) DERIV1, Derivative

To get the first derivative of a data array using a center difference scheme except the two end points in which three points near the boundary are used.

5) DGCOEF, Generalized Coefficients

To calculate the added mass, drag and all the generalized coefficients (A_a, A_d and C_{ij}). This is the heart of the modeling.

The effective coefficients are modeled as:

$$A_a = A_{a1}w_{a1} + A_{a2}w_{a2}$$

$$A_d = A_{d1}w_{d1} + A_{d2}w_{d2}$$

$$A_{a1} = \alpha_1\alpha_2\rho_2/(\alpha_1 + 2\alpha_2/(1 + 3\alpha_1))$$

$$A_{a2} = \alpha_1\alpha_2\rho_1/(\alpha_2 + 2\alpha_1/(1 + 3\alpha_2))$$

$$A_{d1} = 18\ \mu_2\alpha_1/d_1^{\,2}\alpha_2$$

$$A_{d2} = 18\ \mu_1\alpha_2/((1 - \alpha_2/0.8)^2 d_2^{\,2})$$

$$w_{a1} = \alpha_2^{\,na}/(\alpha_1^{\,na} + \alpha_2^{\,na})$$

$$w_{a2} = 1 - w_{a1}$$

$$w_{d1} = \alpha_2^{\,nd}/(\alpha_1^{\,nd} + \alpha_2^{\,nd})$$

$$w_{d2} = 1 - w_{d1}$$

6) DWPDE, Partial Differential Equations.

To evaluate the values of the increments on the column matrix W from the partial differential equations. This is the major part of the McCormack's scheme. In each complete time step this routine will have to be called twice.

7) FNDDT, Δt

To determine the suitable time-step size.

8) FSOFW, Column matrices F and S.

To determine the convective matrix F and the source matrix S.

9) JET, Injection.

To determine the momentum source due to the jet injection.

10) SIZES

To determine the gas bubble and liquid droplet sizes. In the model the sizes were modeled to be functions only of the volume fraction, i.e.

$$d_k = d_{ok} \alpha_k^{\gamma}.$$

Different models for size distributions could be easily adopted here.

11) TAUOFW

To determine the stress tensor τ and its derivative.

12) UOFW

To convert the column matrix W into the physical independent variables, such as α, V_r, V_θ.

V. Summary

A computer program aimed at the phase separation between gas and liquid at zero gravity, induced by vortex motion, is developed. The vortex motion is created by fluid injections. The computer program uses a FORTRAN 77 based code and HP-1000 minicomputer. It is flexible and accepts various input parameters for different flow conditions. Other interaction effects can also be added or modified easily. This program can be used to study the fluid dynamical behavior of the rotational two-phase fluids in a cylindrical tank. It provides a quick/easy sensitivity test on various parameters and thus provides the guidance for the design and use of actual physical systems for handling two-phase fluids

VI. Acknowledgments

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VII. References

1. Yeh, T. T. (1986) "A Vortex-Induced, Gas-Liquid Separation in a Cylindrical Tank at Zero Gravity", NBSIR 86-3322.
2. Drew, D. A. (1983) "Mathematical Modeling of Two-Phase Flow", Annual Review of Fluid Mechanics, Vol. 15.
3. MacCormack, R. W. (1969) "The Effect of Viscosity in Hypervelocity Impact Cratering", AIAA Paper 69-354.
4. Chima, R. V. and Johnson, G. M. (1985) "Efficient Solution of the Euler and Navier-Stokes Equations with a Vectorized Multiple-Grid Algorithm", AIAA Vol. 23, No. 1.
5. Peyret, R. and Taylor, T. D. (1983) "Computational Methods for Fluid Flow" Springer-Verlag, New York.

Appendix A

Code Listing

```
:&GLVM  T=00004 IS ON CR T4   USING 00126 BLKS R=0000

FTN77
$EMA /DATA/,/WWWW/,/COEFF/,/SOURCE/,/FANDS/,/TAU/
$FILES 1,2
      PROGRAM GLVM(,99),(860425.1537)
C
C  THIS PROGRAM WAS DEVELOPED TO STUDY THE FLUID DYNAMICAL BEHAVIOR
C  OF A ROTATIONAL TWO-PHASE FLUIDS(GAS/LIQUID) IN A CYLINDRICAL TANK.
C  THE VORTEX MOTIONS ARE ESTABLISHED BY TANGENTIAL FLUID INJECTION.
C  THE PROGRAM WAS DEVELOPED ORIGINALLY BY T.T. YEH OF NBS
C  IT WAS BASED ON HP'S FORTRAN 77(ANSI 77+MIL-STD-1753)
C
C  WHEN  WHO WHAT
C 8502XX TTY ZE 0-G FUEL TRANSFER, START-UP STAGE.
C               LAX WENDROFF 2-STEP SCHEME (FULL STEP PREDICTION+CORRECTION)
C               WITH NUMERICAL DAMPING FACTOR(NORMALLY SET TO ZERO)
C               CONSERVATION FORM, VARIABLE(AUTO) TIME STEP
C               REAL*8
C               INTERFACIAL FORCES: DRAG,ADDED MASS
C               PUMP CONDITION: MOMENTUM SOURCE BUT NO MASS SOURCE
C 850715 TTY GENERALIZED EQUATIONS AND COEFF. Cij
C 851018 TTY IN ANSI 77 STANDARD( WITH A LITTLE EXCEPTION FOR
C                                   TESTING IN HP-1000)
C
C ***** INTERNAL SUBROUTINES  *****
C          DELA, DERIV1, DGCOEF, DWPDE,FNDDT,FSOFW,
C          INIT, JET, SIZES, TAUOFW and UOFW
C *** MOST OF THE LIST OF NOTATIONS ARE GIVEN IN SUBROUTINE..INIT
C
      CHARACTER NAMR*16, NOTES*72
      INTEGER  I,IOS,J,JTIME(5),K,MM,NPRT,NT
     Z          ,IW,LUP,LUS,NG,NGM1,NGM2
      PARAMETER (MM=101)
      REAL*8 BA(MM,2),DDT,DT,DTMAX,DTMIN,DTPRT,PZERO
     X       ,DW1, T,TMAX,TPRT,VJT, VDR(2)
     Y       ,RJ1,RJ2,TJ1,TJ2,QJ,VJ
     1       ,ALMT,U,V,ALP,P,R,  W,WP,WN,DW,RDP,RH,  F,S
     4       ,BR,BRH,  RHO,MUEF,V18,NA,ND
     5       ,DO,GAMMA,  DAMP,DR
     6       ,TRR,TRA,TAA,RTRR,RTRA,  C,CPA,CD

      COMMON
     Y   /JETS/ RJ1,RJ2,TJ1,TJ2,QJ,VJ
     Z   /CONTP/ IW,LUP,LUS,NG,NGM1,DAMP,DR
     1   /ALPLMT/ ALMT(2)
     2   /COEFF/ C(MM,2,2),CPA(MM,2),CD(MM,2)
     3   /DATA/ U(MM,2),V(MM,2),ALP(MM,2),P(MM),R(MM)
     4   /DRAG1/ RHO(4),MUEF(2),V18(2),NA,ND
     6   /FANDS/ F(MM,5),S(MM,5)
     7   /SOURCE/ BR(MM),BRH(MM)
     8   /TAU/ TRR(MM,2),TRA(MM,2),TAA(MM,2),RTRR(MM,2),RTRA(MM,2)
     9   /WWWW/  W(MM,5),WP(MM,5),WN(MM,5),DW(MM,5),RDP(MM),RH(MM)

      EQUIVALENCE (WN,BA)

C  ******  RHO(1) < RHO(2) (i.e. PHASE-1=GAS, PHASE-2=LIQUID) ******
C     DTPRT          TIME STEP FOR PRINTOUT(AND PLOT)
C

```

```
      LUP=1           ! LU FOR PRINTING DEBUG DATA(=1 TERMINAL)
      LUS=6           ! LU FOR STORING DATA(=6 PRINTER)

    7 FORMAT(2X,A,3I5)
    8 FORMAT(2X,A,3(1PE12.4))
      WRITE(1,7) 'Enter lu for saving data.  D.F.=',LUS
      READ(1,*) LUS

C     KEET JOB TIME FOR FUTURE REFFERENCE
C     CALL EXEC(11,JTIME,JTIME(1))

      IF(LUS .NE. 1 .AND. LUS .NE. 6) THEN
C    ******** Define a file name for storing output ***************
       WRITE(1,'(2A)') 'Enter FILE NAME for saving data.'
       READ(1,'(A)') NAMR
       LUS=90
       OPEN(LUS,FILE=NAMR,IOSTAT=IOS,STATUS='NEW',ERR=999)
      ENDIF

      WRITE(1,'(A)') ' Enter NOTES(<73 CHAR.) for the job'
      READ(1,'(A)') NOTES
      WRITE(LUS,'(3H1   ,A)') NOTES
C     WRITE(LUS,'(5I4)') JTIME

C     To set-up the initial condition.
      CALL INIT
      NGM2=NG-2
      VDR(1)=2.*MUEF(1)/DR**2      ! For determining time step
      VDR(2)=2.*MUEF(2)/DR**2

      T=0.                         ! INITIAL TIME
      THAX=5.
      DTPRT=0.2
      WRITE(1,8) 'Enter INITIAL and FINAL TIMEs. D.F.=',T,TMAX
      READ(1,*) T,TMAX
      WRITE(1,8) 'Enter TIME STEP for output. D.F.=',DTPRT
      READ(1,*) DTPRT

      DTMIN=1.0D-6                 ! SET MINIMUM TIME STEP
      DTMAX=DTPRT

      NT=0                         ! Time step number
      TPRT=T
      NPRT=0
      PZERO=0.D0                   ! Pressure at center core
      DT=DTMIN

   10 NT=NT+1
      CALL UOFW(W,R,NG)

C     PRINT OUT AT SELECTED TIME
      IF(T .GE. TPRT .OR. T .GT. TMAX) THEN
       IF(NT .GT. 1) THEN
        NPRT=NPRT+1
        TPRT=TPRT+DTPRT*(1.+DNINT((T-TPRT)/DTPRT))
   21 FORMAT(1H1,2(A5,I4,A5,1PE .3))
        WRITE(LUP,21) 'NP=',NPRT,'T=',T,'NT=',NT,'DT=',DT
        WRITE(LUS,21) 'NP=',NPRT,'T=',T,'NT=',NT,'DT=',DT

        WRITE(LUS,'(A5,A6,5A11)') 'J','ALP1','U1','U2','V1','V2','P'
```

```
          DO 30 J=1,NG
   30      WRITE(LUS,'(I3,F6.4,5(1PE11.3))')
     1          J,ALP(J,1),U(J,1),U(J,2),V(J,1),V(J,2),P(J)
        ENDIF
       ENDIF

       IF( T .GT. TMAX) GOTO 9999

C      TO SOLVE THE DIFFERENTIAL EQUATIONs
C      USING 2 STEP LAX-WENDROFF SCHEME
C      MacCormack's method.. BACKWARD PREDICTOR, FORWARD CORRECT
C        CENTER DIFFERENCED ON TAU

C      FIRST TO GET THE SPECIAL VARIABLES AND THEIR SPACIAL DERIVATIVEs

       CALL DGCOEF(ALP,NG)                ! DRAG AND GENERIZED COEFF.
       CALL TAUOFW(MUEF,DR,R,NG)          ! STRESS


       VJT=VJ
       IF( T .LT. TJ1 .OR. T .GT. TJ2) VJT=0.   ! NO INJETCTION
       CALL JET(BA,RHO,BR,V,VJT,NG)       ! MOMENTUM SOURCE
       CALL FSOFW(W,BA,R,NG)              ! CONVECTIVE -F AND SOURCE-S

C      DETERMINE THE TIME-STEP SIZE
       IF( NT .GT. 1) THEN
        CALL FNDDT(DT,DR,VDR,NG)
        DT=DT+5.*DTMIN/10.**NT            ! INITIAL INPLUSE TREAMENT
        IF(DT .GE. DTMIN) THEN
         I=DLOG10(DT/DTMIN)               ! ROUND OFF TIME STEP
         DDT=DTMIN*10.**I
         DT=DINT(DT/DDT+0.001)*DDT
         IF( DT .GT. DTMAX) DT=DTMAX
        ELSE
         WRITE(LUP,'(5X,A,1PE13.3)')
     1        'STOP DUE TO TOO SMALL TIME STEP.  DT=',DT
         GOTO 9999
        ENDIF
       ENDIF

C      BACKWARD PREDICTOR

       CALL DWPDE(DW,RDP, DR,DT,RH,NGM1)          ! INCREAMENT

       DO 40 J=1,NGM1
        DO 40 I=1,5
   40    WP(J,I)=0.5*(W(J+1,I)+W(J,I))+DW(J,I)   ! BASE+INCREAM

C      PREDICTION DATA COMPLETED, CONTINUE FOR CORRECTION

       CALL UOFW(WP,RH,NGM1)
       CALL DGCOEF(ALP,NGM1)
       CALL TAUOFW(MUEF,DR,RH,NGM1)
       CALL JET(BA,RHO,BRH,V,VJT,NG)
       CALL FSOFW(WP,BA,RH,NGM1)

C      FORWARD CORRECTION
       CALL DWPDE(DW,P(2), DR,DT,R(2),NGM2)
       P(1)=PZERO
       DO 50 J=1,NGM2
```

```
      P(J+1)=P(J)+(0.25*(RDP(J)+RDP(J+1))+0.5*P(J+1))/R(J)
      DO 50 I=1,5
 50    WN(J+1,I)=.25*(WP(J+1,I)+WP(J,I))+.5*(W(J+1,I)+DW(J,I))
      P(NG)=P(NGM1)

C     2ND STEP (PREDICTION+CORRECTION) COMPLETED

      IF( NT .EQ. 1) GOTO 10
C     Estimation of initial P completed, return to the initial condition
C     and start to advance the program in time.

*************************************************************************
C     DATA W AT THE NEW TIME STEP COMPLETED

C      IMPOSED B.C   #6.4
      DW1=.5*(WN(2,1)+W(2,1))-0.
      ALP(1,1)=(.5*(W(1,5)+W(2,5))-DW1*DT/DR)/RH(1)
      IF(ALP(1,1) .LT. ALMT(1)) ALP(1,1)=ALMT(1)
      IF(ALP(1,1) .GT. ALMT(2)) ALP(1,1)=ALMT(2)
      WN(1,5)=ALP(1,1)*R(1)

      DW1=0.-0.5*(WN(NGM1,1)+W(NGM1,1))
      ALP(NG,1)=(.5*(W(NGM1,5)+W(NG,5))-DW1*DT/DR)/RH(NGM1)
      IF(ALP(NG,1) .LT. ALMT(1)) ALP(NG,1)=ALMT(1)
      IF(ALP(NG,1) .GT. ALMT(2)) ALP(NG,1)=ALMT(2)
      WN(NG,5)=ALP(NG,1)*R(NG)

      WN(1,1)=0.                        ! NO RADIAL VEL.
      WN(1,2)=0.
      WN(NG,1)=0.
      WN(NG,2)=0.
      WN(1,3)=WN(2,3)*WN(1,5)/WN(2,5)*R(1)/R(2) ! OMEGA=CON.
      WN(1,4)=WN(2,4)*(R(1)-WN(1,5))/(R(2)-WN(2,5))*R(1)/R(2)
      IF(IW .EQ. 0) THEN
       WN(NG,3)=WN(NGM1,3)*WN(NG,5)/WN(NGM1,5)*R(NG)/R(NGM1)
       WN(NG,4)=WN(NGM1,4)*(R(NG)-WN(NG,5))/(R(NGM1)-WN(NGM1,5))
     1          *R(NG)/R(NGM1)
      ELSE
       WN(NG,3)=0.                      ! NON-SLIP AT WALL
       WN(NG,4)=0.
      ENDIF

C     ARTIFICIAL DAMPING
      DO 60 I=1,5
       W(1,I)=(1.-DAMP)*WN(1,I)+DAMP*WN(2,I)
       W(NG,I)=(1.-DAMP)*WN(NG,I)+DAMP*WN(NGM1,I)
       DO 60 J=2,NGM1
 60     W(J,I)=(1.-DAMP)*WN(J,I)+DAMP*(WN(J-1,I)+WN(J+1,I)-WN(J,I))

C     SOLUTION FOR THIS TIME STEP COMPLETED


 90   T=T+DT                     ! UPDATE TIME  AND CONTINUE TO THE NEXT STEP
      GOTO 10

999   WRITE(LUP,7) 'OPEN FILE FAILED ON FILE:'
      WRITE(LUP,7) NAMR
      WRITE(LUP,7) 'IOSTAT=',IOS

9999  CONTINUE
```

```
C      CALL EXEC(11,JTIME,JTIME(1))
C      WRITE(LUS,'(35X,5I4)') JTIME
       CLOSE(LUS)

       END

*************************************************************************
$EMA /DATA/,/WWWW/,/SOURCE/
       SUBROUTINE INIT ,(860425.1537)

C      TO SET-UP THE INITIAL CONDITIONS
       INTEGER I,IOS,ITLOG,IVTX,J,K,MM
      Z          ,ITIME,IW,LUP,LUS,NG,NGM1
       PARAMETER (MM=101)
       CHARACTER NAMR*16
       REAL*8 ALMT,  W,WP,WN,DW,RDP,RH
      1         ,U,V,ALP,P,R,   F,S
      4         ,RHO,MUEF,V18,NA,ND,   DO,GAMMA
      7         ,BR,BRH, DAMP,DR
      Y         ,RJ1,RJ2,TJ1,TJ2,QJ,VJ
      X         ,A2,DELA,DEN1D2,D1,D2,PI,RE,RMIN,RTANK
      X         ,DS,LS,VS,TS,PS
      X         ,ALP10,OMEGA,RPEAK,RJB,VPEAK
      X         ,EVF(2),VT,MU(2),MU1D2

       COMMON
      1   /ALPLMT/ ALMT(2)
      Y   /JETS/ RJ1,RJ2,TJ1,TJ2,QJ,VJ
      Z   /CONTP/ IW,LUP,LUS,NG,NGM1,DAMP,DR
      3   /DATA/ U(MM,2),V(MM,2),ALP(MM,2),P(MM),R(MM)
      4   /DPAG1/ RHO(4),MUEF(2),V18(2),NA,ND
      5   /DSIZE/ DO(2),GAMMA(2)
      7   /SOURCE/ BR(MM),BRH(MM)
      9   /WWWW/  W(MM,5),WP(MM,5),WN(MM,5),DW(MM,5),RDP(MM),RH(MM)

       DATA PI/3.141596D0/


C  ****** NOTES:  PHASE-1=GAS, PHASE-2=LIQUID  ******
C     RHO              DENSITY, RHO(1) < RHO(2)
C     ALMT             LIMIT VALUES FOR ALP1, ALM(1)<ALP1<ALM(2)
C     DAMP             NUMERICAL DAMPING FACTOR, NORMALLY =0.
C     DS,LS,VS,TS      DENSITY,LENGTH,VELOCITY AND TIME SCALES
C     RE               REYNOLDS #=Vs*RTANK*RHO(2)/MU(2)
C     RTANK            TANK RADIUS
C     IVTX             TYPE OF INITIAL FLOW.
C                      (0=Rest,1=Pure rotation,2=H.O.3=GIT Vertex)
C     RPEAK,VPEAK      VORTEX PARAMETERS
C     OMEGA            PURE ROTATION. V=OMEGA*r
C     EVF,MUEF         EFFECTIVE VISCOSITY,MUEF=(1+EVF)*MU
C     DO,GAMMA         DIA. PARAMETERS: D=DO*ALF**GAMMA
C     RJ1,RJ2,TJ1,     TO DEFINE JET SIZE, PUMPING TIME,
C     TJ2,QJ,VJ        VOLUME FLOW RATE AND INJECTION MEAN SPEED
C

 5     FORMAT(2X,A,2(X,A))
 7     FORMAT(2X,A,3I5)
 8     FORMAT(2X,A,3(1PE12.4))
 9     FORMAT(A25,2(1PE15.4))
 92    FORMAT(X,7(1PE11.4))
```

```
C      DEFINE THE PARAMETERS FOR THE PROBLEM.

       RTANK=1.D0                        ! COULD BE SET TO 1 (M)
       RHO(2)=1.000D+3                   !    "  (KG/M**3)
       MU(2)=1.514D-3                    !    "  (KG/M-S)

C      WRITE(1,8) 'ENTER RTANK(M) OR DEFAULT  ',RTANK
C      READ(1,*) RTANK

C         The values of RTANK,RHO(2),and MU(2) could all be set to 1, since
C      the length and density scales are based on RTANK,and RHO(2) and the
C      value of the viscosity MU(2) can be combined into and specified by the
C      Reynolds number RE.  Thus all characteristic scales(LS,VS,TS,and DS)
C      are fixed after RE is given.

       RE=1.0D5                          ! LS**2/(MU*TS)=LS*VS/MU
       WRITE(1,8) 'enter Reynolds no.,RE.  D.F.=',RE
       READ(1,*) RE
       LS=RTANK                          ! LENGTH SCALE  (M)
       DS=RHO(2)                         ! DENSITY SCALE (KG/M**3)
       VS=RE*MU(2)/RHO(2)/LS             ! VELOCITY SCALE(M/S)
       TS=LS/VS                          ! TIME SCALE (S)
       PS=DS*VS**2                       ! PRESSURE SCALE


C      AFTER THIS POINT ALL VARIABLES ARE BAESED ON THE CHARA. SCALES
C      i.e. ALL VARIABLES ARE DIMENSIONLESS


       DEN1D2=1.293D0/1.000D3            ! D1/D2
       MU1D2=1.71D-5/1.514D-3            ! MU1/MU2

       RHO(2)=RHO(2)/DS
       MU(2)=MU(2)/(DS*LS*VS)            ! =1/RE
       RHO(1)=DEN1D2*RHO(2)
       MU(1)=MU1D2*MU(2)

       ALMT(1)=0.00010                   ! MIN. OF ALP1
       ALMT(2)=0.99999                   ! MAX. OF ALP1
       DO(1)=1.D-2/LS                    ! GAS DIAMETER at ALP1=1
       DO(2)=1.D-2/LS                    ! LIQUID DIAMETER at ALP2=1
       GAMMA(1)=2.D-1                    !
       GAMMA(2)=2.D-1
       EVF(2)=1.D3                       ! TURB.+PHASE-DISPERSION EFFECTS
       EVF(1)=DEN1D2/MU1D2*EVF(2)        ! MODEL
       DAMP=0.0                          ! NUMERICAL DAMPING FACTOR(e.g =.2)
       NA=4.                             ! WEIGHTING EXP. FOR ADM
       ND=4.                             ! WEIGHTING EXP. FOR DRAG

       WRITE(1,8) 'Enter DENSITY and VISCOSITY ratios.'
       WRITE(1,8) 'D.F.=',DEN1D2,MU1D2
       READ(1,*) DEN1D2,MU1D2

       WRITE(1,8) 'Enter BASE DIAMETERS: DO1,DO2'
       WRITE(1,8) 'D.F.=',DO
       READ(1,*) DO

       WRITE(1,8) 'Enter SIZE EXPONENT: GAMMA1,GAMMA2'
       WRITE(1,8) 'D. F. =',GAMMA
```

```
      READ(1,*) GAMMA

      WRITE(1,8) 'Enter weighting exponent: NA,ND. D.F.=',NA,ND
      READ(1,*) NA,ND

      WRITE(1,8) 'Enter GAS VOLUME FRACTION limits:ALMT1,ALMT2.'
      WRITE(1,8) 'D.F.=',ALMT
      READ(1,*) ALMT

      WRITE(1,8)  'Enter eddy visicesity factor. D.F =',EVF
      READ(1,*) EVF

      IW=1
      WRITE(1,7) 'Enter wall conditien,1=nonslip,0=slip. D.F.',IW
      READ(1,*) IW

      WRITE(1,8) 'Enter numerical damping factor. D.F.=',DAMP
      READ(1,*) DAMP

      DO 10 K=1,2
       V18(K)=18.*MU(K)
 10    MUEF(K)=MU(K)*(1.+EVF(K))   ! EFFECTIVE VISCOSITY FOR STRESS

      RHO(3)=RHO(1)*RHO(2)
      RHO(4)=RHO(1)-RHO(2)

      RMIN=0.1                     ! MINIMUM FLOW RADIUS IN THE TANK
      NG=101                       ! # OF GRID POINTs USED
      NGM1=NG-1
      DR=(1.-RMIN)/NGM1

C    Initial cleanning-up.
      DO 15 J=1,MM
       DO 15 K=1,6
        W(J,K)=0.D0
 15     U(J,K)=0.D0

C     MOMENTUM SOURCE, JET CONDITIONS
      RJ1=8.5D-1
      RJ2=9.5D-1
      VJ=10.                       ! TANGENTIAL INJECTION SPEED
      TJ1=0.
      TJ2=10.

      WRITE(1,8) 'Enter JET SIZE defined by RJ1,RJ2. D.F.=',RJ1,RJ2
      READ(1,*) RJ1,RJ2
      WRITE(1,8) 'Enter INJECTION SPEED AND TIME RANGE, VJ,T1,T2'
      WRITE(1,8) 'D.F.=',VJ,TJ1,TJ2
      READ(1,*) VJ,TJ1,TJ2
      QJ=(RJ2-RJ1)*VJ              ! JET VOLUME FLOW RATE

      DO 20 J=1,NG
       R(J)=RMIN+(J-1)*DR
       RH(J)=R(J)+0.5*DR
       BR(J)=DELA((R(J)),DR,RJ1,RJ2)/(2.*PI)      ! JET DISTRIBUTION
 20    BRH(J)=DELA((RH(J)),DR,RJ1,RJ2)/(2.*PI)    ! PER RADIAN


C     SETUP INITIAL CONDITIONS
      IVTX=0
```

```
      OMEGA=0.
      VPEAK=0.
      RPEAK=RMIN

      NAMR='Simple vortex'
      WRITE(1,5) 'Enter data FILE NAME for initial cond., if any;'
      WRITE(1,5) 'D.F.=',NAMR
      READ(1,'(A)') NAMR

      IF( NAMR .NE. ',' .AND. NAMR .NE. 'Simple vortex') THEN
C     INITIAL CONDITION FROM A GIVEN FILE NAMR.
       OPEN(99,FILE=NAMR,IOSTAT=IOS,STATUS='OLD',ERR=299)
       DO 25 J=1,NG                    ! INITIAL VALUES FROM FILE NAMR
   25   READ(99,*) ..,ALP(J,1),U(J,1),U(J,2),V(J,1),V(J,2)
       CLOSE(99)

      ELSE
C     TO DEFINE INITIAL CONDITION.
       ALP10=2.5D-1                    ! INITIAL GAS VOL. FRACTION
       WRITE(1,8) 'Enter initial value of alp1. D.F.=',ALP10
       READ(1,*) ALP10

       WRITE(1,7) 'Enter type of vortex: 0=At rest,1=pure rotation'
       WRITE(1,7) '2=H.O.,3=GIT.  D.F.=',IVTX
       READ(1,*) IVTX
       IF(IVTX .GT. 0) THEN
        IF(IVTX .GT. 1) THEN
         WRITE(1,8) 'Enter PEAK VEL  and LOCATION for classic vortex'
         WRITE(1,8) 'D.F.=',VPEAK,RPEAK
         READ(1,*) VPEAK,RPEAK
         IF(RPEAK .LE. 0.) RPEAK=1.     ! SINGULAR AT ZERO
        ELSE
         WRITE(1,8) 'Enter CIRCULAR SPEED(rad./unit time).D.F.='
     1               ,OMEGA
         READ(1,*) OMEGA
        ENDIF
       ENDIF

       DO 30 J=1,NG
        ALP(J,1)=ALP10
        VT=OMEGA*R(J)                   ! PURE ROTATION
        IF(IVTX .GT. 0) THEN
         RJB=R(J)/RPEAK
         IF( IVTX .EQ. 1) THEN          ! H.O. VORTEX
          VT=VT+1.398*VPEAK/RJB*(1.-DEXP(-1.25643*RJB**2))
         ELSE                           ! G.I.T. VORTEX
          VT=VT+VPEAK*RJB*DEXP((1.-RJB**2)/2.)
         ENDIF
        ENDIF
        DO 30 K=1,2
         IF(IW .EQ. 0) THEN
          V(J,K)=VT                     ! NO WALL
         ELSE
          V(J,K)=VT*(1.-R(J))**0.1      ! BOUNDARY LAYER
         ENDIF
   30    U(J,K)=0.                      ! NO RADIAL VEL.

      ENDIF

      DO 40 J=1,NG                      ! FORM W FOR NUMERICAL CAL.
```

```
      ALP(J,2)=1.-ALP(J,1)
      P(J)=0.D0
      W(J,5)=R(J)*ALP(J,1)
      DO 40 K=1,2
        W(J,K)=ALP(J,K)*U(J,K)*R(J)
40      W(J,K+2)=ALP(J,K)*V(J,K)*R(J)


C     PRINTOUT PARAMETERS
      WRITE(LUS,5) 'INITIAL CONDITION FILE:',NAMR
      WRITE(LUS,5) '**DIMENSION UNITS ARE IN MKS**'
      WRITE(LUS,9) 'DENSITY SCALE(kg/m3)',DS
      WRITE(LUS,9) 'LENGTH SCALE=RTANK,(m)',LS
      WRITE(LUS,9) 'VELOCITY SCALE(m/s)',VS
      WRITE(LUS,9) 'TIME SACLE(s)',TS
      WRITE(LUS,9) 'PRESSURE SCALE(Pa)',PS
      WRITE(LUS,*)
      WRITE(LUS,9) 'Reynolds number, Re',RE
      WRITE(LUS,9) 'Jet size, RJ1,RJ2',RJ1,RJ2
      WRITE(LUS,9) 'Tangential jet, QJ,VJ',QJ,VJ
      WRITE(LUS,9) 'Injection time,TJ1,TJ2',TJ1,TJ2

      WRITE(LUS,'(/33X,"PHASE-1",8X,"PHASE-2")')
      WRITE(LUS,9) 'Density',RHO(1),RHO(2)
      WRITE(LUS,9) 'Viscosity',MU
      WRITE(LUS,9) 'Eddy viscosity factor',EVF
      WRITE(LUS,9) 'Base dia.',DO
      WRITE(LUS,9) 'Size exp.',GAMMA
      WRITE(LUS,9) 'Phase limits',ALMT(1),1.-ALMT(2)
      WRITE(LUS,*)
      WRITE(LUS,*)
      WRITE(LUS,'(" OTHER CONSTANS: IW,IVTX,NA,ND,DAMP,VPEAK,RPEAK",
     1            ",OMEGA,D1/D2,MU1/MU2")')
      WRITE(LUS,'(2I3,[illegible].0,F5.2)') IW,IVTX,NA,ND,DAMP
      WRITE(LUS,92) VPEAK,RPEAK,OMEGA,DEN1D2,MU1D2
      WRITE(LUS,*)
      WRITE(LUS,'(I10,F10.2)') NG,RMIN

      RETURN

299   WRITE(LUP,5) 'OPEN FILE FAILED ON INPUT FILE:',NAMR
      WRITE(LUP,7) 'IOSTAT=',IOS
      STOP 111
      END

*********************************************************************
      REAL*8 FUNCTION DELA(R,DR,RJ1,RJ2),(860423.1537)
C     TO DETERMINE THE EFFECTIVE NOZZLE SIZE AT EACH GRID LOCATI
C     THE SIZE IS IN THE FRACTION OF GRID SIZE DR (i.e. 0(DELA(1)

      REAL*8 R,DR,RJ1,RJ2,  R1,R2

      R1=R-0.5*DR
      R2=R1+DR
      DELA=0.D0
      IF(R1 .GE. RJ2 .OR. R2  LE. RJ1) RETURN
      IF(R1 .LT. RJ1) R1=RJ1
      IF(R2 .GT. RJ2) R2=RJ2
      DELA=(R2-R1)/DR
      RETURN
```

```
      END

*****************************************************************
      SUBROUTINE DERIV1(Y,DY,DX,N2),(860425.1537)
C     GET 1ST DERIVATIVE, USING CENTERED DIFFERENCE
      REAL*8 DX,Y(1),DY(1),C
      EMA Y,DY

      C=5.D-1/DX
      DO 10 J=2,N2-1
  10   DY(J)=C*(Y(J+1)-Y(J-1))
*V    CALL DWSUB(Y(3),1,Y,1,DY(2),1,N2-2)

      DY(1)=(Y(2)-Y(1))/DX      ! BASED ON 3-END PTS
      DY(N2)=(Y(N2)-Y(N2-1))/DX
      DY(1)=2.*DY(1)-DY(2)
      DY(N2)=2.*DY(N2)-DY(N2-1)

*V    CALL DWSMY(5.D-1/DX,DY,1,DY,1,N2)

      RETURN
      END


*****************************************************************
$EMA /COEFF/
      SUBROUTINE DGCOEF(ALP,N2),(860425.1537)
C     CALCULATE THE DRAG, ADDED MASS AND GENERIZED COEFF.

      INTEGER J,MM,N2
      PARAMETER (MM=101)
      REAL*8 ALP(MM,2)
      EMA ALP

      REAL*8  C,CPA,CD,  RHO,MUEF,V18,NA,ND
     X        ,AA,AA1,AA2,AD,AD1,AD2,A1,A2,A12,DA12,DB2, D1,D2,WT1,WT2,X
      COMMON
     1   /COEFF/ C(MM,2,2),CPA(MM,2),CD(MM,2)
     4   /DRAG1/ RHO(4),MUEF(2),V18(2),NA,ND

      DO 50 J=1,N2
       A1=ALP(J,1)
       A2=ALP(J,2)
       A12=A1*A2

C     TO GET DRAG COEFF. AD
       CALL SIZES(D1,D2,A1)

       AD=V18(2)*A1/(A2*D1*D1)        ! =AD1 IF A2).78

       IF( A2 .LT. .78) THEN
        AD2=V18(1)*A2/((1.-A2/.8)*D2)**2
        IF(AD2 .LT. AD) THEN
         WT1=A2**ND
         WT2=A1**ND
         AD=(AD*WT1+AD2*WT2)/(WT1+WT2)
        ENDIF
       ENDIF

C     ADDED MASS COEFF. AA
```

```
       AA1=A12*RHO(2)/(A1+A2/(.5+1.5*A1))
       AA2=A12*RHO(1)/(A1/(.5+1.5*A2)+A2)

       WT1=A2**NA
       WT2=A1**NA
       AA=(AA1*WT1+AA2*WT2)/(WT1+WT2)

C    THE GENERIZED COEFF. CPA,C,AND CD
       DB2=A12*RHO(3)+AA*(RHO(1)*A1+RHO(2)*A2)
       CPA(J,1)=A1*(A12*RHO(2)+AA)/DB2
       CPA(J,2)=A2*(A12*RHO(1)+AA)/DB2
       C(J,1,2)=AA/DB2
       C(J,2,1)=C(J,1,2)
       C(J,1,1)=A2*RHO(2)/DB2+C(J,1,2)
       C(J,2,2)=A1*RHO(1)/DB2+C(J,2,1)
       CD(J,1)=-A2*RHO(2)*AD/DB2
       CD(J,2)=A1*RHO(1)*AD/DB2

   50 CONTINUE

      RETURN
      END

*********************************************************************
$EMA /DATA/,/FANDS/,/TAU/,/COEFF/
      SUBROUTINE DWPDE(DW,RDP,  DR,DT,RR,N2),(860425.1537)
C
C    TO GET DW OF THE PDEs
C
      INTEGER J,JP1,K,KP2,MM,N2
      PARAMETER (MM=101)
      REAL*8 DW(MM,5),RDP(MM), DR,DT,RR(MM)
      EMA DW,RDP,RR

C       NOTES: COEFF. C =C*ALP WHEN THIS IS CALLED
      REAL*8 C,CPA,CD, U,V,ALP,P,R,  F,S
     8       ,TRR,TRA,TAA,RTRR,RTRA
     4       ,RHO,MUEF,V18,NA,ND
     X       ,ALP1,ALP2,CP1,CP2,DTDR,DW1,DW1DT,G1,G2,HDT,VT,WJ1,WJ3,WJ4

      COMMON
     1  /COEFF/ C(MM,2,2),CPA(MM,2),CD(MM,2)
     3  /DATA/ U(MM,2),V(MM,2),ALP(MM,2),P(MM),R(MM)
     4  /DRAG1/ RHO(4),MUEF(2),V18(2),NA,ND
     6  /FANDS/ F(MM,5),S(MM,5)
     8  /TAU/ TRR(MM,2),TRA(MM,2),TAA(MM,2),RTRR(MM,2),RTRA(MM,2)

      DTDR=DT/DR
      HDT=0.5*DT

      DO 10 J=1,N2+1                  ! CHANGE C TO ALP*C
       C(J,1,1)=ALP(J,1)*C(J,1,1)
       C(J,2,1)=ALP(J,2)*C(J,2,1)
       C(J,1,2)=ALP(J,1)*C(J,1,2)
   10  C(J,2,2)=ALP(J,2)*C(J,2,2)

      DO 20 J=1,N2
       JP1=J+1
       DW(J,5)=DTDR*(-F(JP1,5)+F(J,5))+HDT*(S(JP1,5)+S(J,5))
       DO 25 K=1,2
```

```
         G1=.5*(C(JP1,K,1)+C(J,K,1))
         G2=.5*(C(JP1,K,2)+C(J,K,2))
         DW(J,K)=DTDR*(-F(JP1,K)+F(J,K)+G1*(RTRR(JP1,1)-RTRR(J,1))
     1            +G2*(RTRR(JP1,2)-RTRR(J,2)))
     2            +HDT*(S(JP1,K)+S(J,K))
         KP2=K+2
   25    DW(J,KP2)=DTDR*(-F(JP1,KP2)+F(J,KP2)
     1            +G1*(RTRA(JP1,1)-RTRA(J,1))
     2            +G2*(RTRA(JP1,2)-RTRA(J,2)))
     3            +HDT*(S(JP1,KP2)+S(J,KP2))

C      DP FOR PRESSURE CORRECTION
        CP1=0.5*(CPA(J,1)+CPA(JP1,1))
        CP2=0.5*(CPA(J,2)+CPA(JP1,2))

        IF( -DW(J,1) .GT. DW(J,2)) DW(J,1)=-DW(J,2)   ! DP)=0
        RDP(J)=(DW(J,1)+DW(J,2))/(CP1+CP2)
        DW(J,1)=DW(J,1)-CP1*RDP(J)

        DW(J,2)=-DW(J,1)
   20   RDP(J)=RDP(J)/DT

       RETURN
       END

**************************************************************************
$EMA /COEFF/,/DATA/,/WWWW/
       SUBROUTINE FNDDT(DT,DR,VDR,NG),(860425.1537)

C      DETERMINE THE TIME-STEP SIZE

       INTEGER I,J,LUP,MM,NG
       PARAMETER (MM=101)
       REAL*8 DT,DR,VDR(2)

       REAL*8  C,CPA,CD,  RHO,MUEF,V18,NA,ND
     9        ,W,WP,WN,DW,RDP,RH, U,V,ALP,P,R
     X        ,DUM1,DUM2

       COMMON
     2  /COEFF/ C(MM,2,2),CPA(MM,2),CD(MM,2)
     3  /DATA/ U(MM,2),V(MM,2),ALP(MM,2),P(MM),R(MM)
     4  /DRAG1/ RHO(4),MUEF(2),V18(2),NA,ND
     9  /WWWW/  W(MM,5),WP(MM,5),WN(MM,5),DW(MM,5),RDP(MM),RH(MM)
       DATA LUP/1/

       DUM1=0.
       DO 10 J=1,NG
        DUM2=-CD(J,1)                                     ! 1/St
     1          +DABS(U(J,1))/DR                          ! CONVECTIVE
     2          +VDR(1)*ALP(J,1)*C(J,1,1)                 ! VDR=2*MUEF/DR**2
     3          +VDR(2)*ALP(J,2)*C(J,1,2)                 ! CROSS VISICOSITY
        IF(DUM1 .LT. DUM2) DUM1=DUM2
        DUM2=CD(J,2)+DABS(U(J,2))/DR
     1          +VDR(2)*ALP(J,2)*C(J,2,2)
     2          +VDR(1)*ALP(J,1)*C(J,2,1)
        IF(DUM1 .LT. DUM2) DUM1=DUM2
   10  CONTINUE

C      FIND THE MAXIMUN OF DW
```

```
*V    CALL DWMAX(I,DW,1,NG)                      ! VECTOR OPERATION
*V    DUM1=DABS(DW(I,1))
*V    CALL DWMAX(J,DW(1,2),1,NG)
*V    DUM2=DABS(DW(J,2))
      DT=1.D0/DUM1

      RETURN
      END

*****************************************************************
$EMA /DATA/,/FANDS/,/TAU/,/COEFF/
      SUBROUTINE FSOFW(W,BA,RR,N2),(860425.1537)
C
C     CALCULATE THE CONVECTIVE-F AND SOURCE-S TERMS
C
      INTEGER J,K,KP2,L,MM,N2
      PARAMETER (MM=101)
      REAL*8 U,V,ALP,P,R,   F,S,  RHO,MUEF,V18,NA,ND
     2       ,TRR,TRA,TAA,RTRR,RTRA,  C,CPA,CD
     X       ,ALPD(2),RDU,RDV

      REAL*8 W(MM,5),BA(MM,2),RR(1)
      EMA W,BA,RR

      COMMON
     1  /COEFF/ C(MM,2,2),CPA(MM,2),CD(MM,2)
     3  /DATA/ U(MM,2),V(MM,2),ALP(MM,2),P(MM),R(MM)
     4  /DRAG1/ RHO(4),MUEF(2),V18(2),NA,ND
     6  /FANDS/ F(MM,5),S(MM,5)
     8  /TAU/ TRR(MM,2),TRA(MM,2),TAA(MM,2),RTRR(MM,2),RTRA(MM,2)

*V    CALL DWMPY(W,1,U,1,F,1,2*MM)
*V    CALL DWMPY(W(1,3),1,U,1,F(1,3),1,2*MM)
*V    CALL DWMOV(W,1,F(1,5),1,MM)

      DO 20 J=1,N2
       F(J,5)=W(J,1)
       S(J,5)=0.

       RDU=RR(J)*(U(J,1)-U(J,2))
       RDV=RR(J)*(V(J,1)-V(J,2))
       DO 20 K=1,2
        KP2=K+2
        F(J,K)=W(J,K)*U(J,K)
        F(J,KP2)=W(J,KP2)*U(J,K)

        S(J,K)=ALP(J,K)*(V(J,K)**2+CD(J,K)*RDU-C(J,K,1)*TAA(J,1)
     1                   -C(J,K,2)*TAA(J,2))
   20   S(J,KP2)=ALP(J,K)*(-U(J,K)*V(J,K)+CD(J,K)*RDV+C(J,K,1)*(BA(J,1)
     1                     +TRA(J,1))+C(J,K,2)*(BA(J,2)+TRA(J,2)))

      RETURN
      END


*****************************************************************
      SUBROUTINE JET(BA,RHO,BR,V,VJ,NG)
C
C     INJECTION MOMENTUM SOURCE
C
```

```
      PARAMETER (MM=101)
      REAL*8 BA(MM,2),RHO(1),BR(1),V(MM,2),VJ
      EMA BA,BR,V

      DO 10 J=1,N2
       IF( BR(J) .GT. 0.) THEN
        Q=BR(J)*VJ                              ! Flow rate/radian
        BA(J,1)=Q*(VJ-V(J,1))*RHO(1)            ! NET MOMENTUM GAIN
        BA(J,2)=Q*(VJ-V(J,2))*RHO(2)
       ELSE
        BA(J,1)=0.D0
        BA(J,2)=0.D0
       ENDIF
   10 CONTINUE
      RETURN
      END

**********************************************************************
      SUBROUTINE SIZES(D1,D2,ALP1),(860425.1537)
C      TO DETERMINE THE PARTICLE DIAMETERS
C
      REAL*8 D1,D2,ALP1,  DO,GAMMA
      COMMON /DSIZE/ DO(2),GAMMA(2)

      D1=DO(1)*ALP1**GAMMA(1)
      D2=DO(2)*(1.0D0-ALP1)**GAMMA(2)

      RETURN
      END

**********************************************************************
$EMA /DATA/,/TAU/
      SUBROUTINE TAUOFW(MU,DR,RR,N2),(860425.1537)
C     STRESSES AND THEIR DERIVATIVES
C
      REAL*8 MU(2),DR,RR(1)
      EMA RR

      PARAMETER (MM=101)
      REAL*8 U,V,ALP,P,R
     2       ,TRR,TRA,TAA,RTRR,RTRA
     X       ,TAMU,TMU

      COMMON
     3  /DATA/ U(MM,2),V(MM,2),ALP(MM,2),P(MM),R(MM)
     8  /TAU/ TRR(MM,2),TRA(MM,2),TAA(MM,2),RTRR(MM,2),RTRA(MM,2)

      DO 50 K=1,2
       CALL DERIV1(U(1,K),TRR(1,K),DR,N2)                     ! dU/dR
*V     CALL DWMPY(ALP(1,K),1,TRR(1,K),1,TRR(1,K),1,N2)        ! *ALP
*V     CALL DWSMY(2.*MU(K),TRR(1,K),1,TRR(1,K),1,N2)          ! *2*MU=TRR
*V     CALL DWDIV(V(1,K),1,RR,1,TAA(1,K),1,N2)' V/R
       DO 10 J=1,N2
   10   TAA(J,K)=V(J,K)/RR(J)
       CALL DERIV1(TAA(1,K),TRA(1,K),DR,N2)                   ! d()/dR

       TMU=2.*MU(K)
       DO 20 J=1,N2
        TAMU=TMU*ALP(J,K)
        TRR(J,K)=TAMU*TRR(J,K)
```

```
          TRA(J,K)=MU(K)*ALP(J,K)*RR(J)*TRA(J,K)
          TAA(J,K)=TAMU*U(J,K)/RR(J)
          RTRR(J,K)=RR(J)*TRR(J,K)
   20     RTRA(J,K)=RR(J)*TRA(J,K)

*V       CALL DWMPY(RR,1,TRA(1,K),1,TRA(1,K),1,N2)          ! *R
*V       CALL DWMPY(ALP(1,K),1,TRA(1,K),1,TRA(1,K),1,N2)    ! *ALP
*V       CALL DWSMY(MU(K),TRA(1,K),1,TRA(1,K),1,N2)         ! *MU*TRA

*V       CALL DWDIV(U(1,K),1,RR,1,TAA(1,K),1,N2)! U/R
*V       CALL DWMPY(ALP(1,K),1,TAA(1,K),1,TAA(1,K),1,N2)    ! *ALP
*V       CALL DWSMY(2.*MU(K),TAA(1,K),1,TAA(1,K),1,N2)      ! *2*MU*TAA

*V       CALL DWMPY(RR,1,TRR(1,K),1,RTRR(1,K),1,N2)         ! R*TRR
*V       CALL DWMPY(RR,1,TRA(1,K),1,RTRA(1,K),1,N2)         ! R*TRA
   50  CONTINUE
       RETURN
       END

*****************************************************************************
$EMA /DATA/
       SUBROUTINE UOFW(W,RR,N2),(860425.1537)
C      CONVERTS W  TO THE INDEPENDENT VARIABLES(U,V,ALP)

       PARAMETER (MM=101)
       REAL*8 W(MM,5),RR(MM)
       EMA W ,RR

       REAL*8 ALMT,  U,V,ALP,P,R
      X        ,WJ6
       COMMON
      X  /ALPLMT/ ALMT(2)
      3  /DATA/ U(MM,2),V(MM,2),ALP(MM,2),P(MM),R(MM)

*V     CALL DWDIV(W(1,5),1,RR,1,ALP,1,N2)

C      CHECK VOLUME FRACTION  & FLOW DIRECTIONS
       DO 50 J=1,N2
       ALP(J,1)=W([illegible])/RR(J)
        IF(ALP(J,1) .LE. ALMT(1) .OR. ALP(J,1) .GE. ALMT(2)) THEN
         IF(ALP(J,1) .LE. ALMT(1)) THEN
          ALP(J,1)=ALMT(1)
          W(J,3)=W(J,4)*ALMT(1)/(1.D0-ALMT(1))
         ELSE
          ALP(J,1)=ALMT(2)
          W(J,4)=W(J,3)*(1.D0-ALMT(2))/ALMT(2)
         ENDIF
         W(J,1)=0.D0
         W(J,2)=0.D0
         W(J,5)=ALP(J,1)*RR(J)

        ENDIF

        ALP(J,2)=1.D0-ALP(J,1)

        IF( W(J,2) .LT. 0.) THEN        ! PHASE-2 DOES NOT MOVE IN
         W(J,2)=0.
         W(J,1)=0.
        ENDIF

```

```
          U(J,1)=W(J,1)/W(J,5)
          V(J,1)=W(J,3)/W(J,5)
         WJ6=RR(J)*ALP(J,2)
          U(J,2)=W(J,2)/WJ6
          V(J,2)=W(J,4)/WJ6
   50    CONTINUE

*V       CALL DWDIV(W,1,W(1,5),1,U,1,N2)
*V       CALL DWDIV(W(1,3),1,W(1,5),1,V,1,N2)
*V       CALL DWMPY(ALP(1,2),1,RR,1,W(1,6),1,N2)
*V       CALL DWDIV(W(1,2).1,W(1,6),1,U(1,2),1,N2)
*V       CALL DWDIV(W(1,4),1,W(1,6),1,V(1,2),1,N2)

         RETURN
         END
```

Exhibit A

A Sample Input

```
:GLVM
  Enter lu for saving data.  D.F.=    6
90
Enter FILE NAME for saving data.
TS153::LB
 Enter NOTES(<73 CHAR.) for the job
SAMPLE RUN OF TEST 153
  enter Reynolds no.,RE.  D.F.=  1.0000E+05
.
  Enter DENSITY and VISCOSITY ratios.
  D.F.=  1.2930E-03  1.1295E-02
..
  Enter BASE DIAMETERS: D01,D02
  D.F.=  1.0000E-02  1.0000E-02
..
  Enter SIZE EXPONENT: GAMMA1,GAMMA2
  D. F. =  2.0000E-01  2.0000E-01
..
  Enter weighting exponent: NA,ND. D.F.=  4.0000E+00  4.0000E+00
..
  Enter GAS VOLUME FRACTION limits:ALMT1,ALMT2.
  D.F.=  1.0000E-04  9.9999E-01
..
  Enter eddy visicosity factor. D.F.=  1.1448E+02  1.0000E+03
1000,1000
  Enter wall condition,1=nonslip,0=slip. D.F.    1
.
  Enter numerical damping factor. D.F.=  0.0000E+00
.
  Enter JET SIZE defined by RJ1,RJ2. D.F.=  8.5000E-01  9.5000E-01
..
  Enter INJECTION SPEED AND TIME RANGE, VJ,T1,T2
  D.F.=  1.0000E+01  0.0000E+00  1.0000E+01
1,0,1
  Enter data FILE NAME for initial cond . if any:
  D.F.= Simple vortex
.
  Enter initial value of alp1. D.F.=  2.5000E-01
.
  Enter type of vortex: 0=At rest,1=pure rotation
  2=H.O.,3=GIT.  D.F.=    0
.
  Enter INITIAL and FINAL TIMEs. D.F.=  0.0000E+00  5.0000E+00
0,.01
  Enter TIME STEP for output. D.F.=  2.0000E-01
.01
1  NP=   1   T=0.000E+00  NT=   2  DT=1.000E-06
1  NP=   2   T=1.200E-02  NT=   6  DT=3.000E-03
:
```

Exhibit B

A Sample Output

```
TS153  T=00004 IS ON CR LB   USING 00024 BLKS R=0000

1  SAMPLE RUN OF TEST 153
   INITIAL CONDITION FILE: .
   **DIMENSION UNITS ARE IN MKS**
      DENSITY SCALE(kg/m3)       1.0000E+03
   LENGTH SCALE=RTANK.(m)        1.0000E+00
       VELOCITY SCALE(m/s)       1.5140E-01
             TIME SACLE(s)       6.6050E+00
        PRESSURE SCALE(Pa)       2.2922E+01

       Reynolds number, Re       1.0000E+05
         Jet size, RJ1,RJ2       8.5000E-01      9.5000E-01
     Tangential jet, QJ,VJ       1.0000E-01      1.0000E+00
   Injection time,TJ1,TJ2        0.0000E+00      1.0000E+00

                                    PHASE-1         PHASE-2
                   Density       1.2930E-03      1.0000E+00
                 Viscosity       1.1295E-07      1.0000E-05
     Eddy viscosity factor       1.0000E+03      1.0000E+03
                 Base dia.       1.0000E-02      1.0000E-02
                 Size exp.       2.0000E-01      2.0000E-01
              Phase limits       1.0000E-04      1.0014E-05


  OTHER CONSTANS: IW,IVTX,NA,ND,DAMP,VPEAK,RPEAK,OMEGA,D1/D2,MU1/MU
   1  0  4.  4. 0.00
   0.0000E+00 1.0000E-01 0.0000E+00 1.2930E-03 1.1295E-02

        101        .10
1  NP=    1    T=0.000E+00   NT=    2   DT=1.000E-06
     J  ALP1       U1          U2          V1          V2          P
     1 .2500  0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00
     2 .2500  0.000E+00  0.000E+00  0.000E+00  0.000E+00  0.000E+00
          --------- OUTPUT IN THE BETWEEN OMITTED -----
    97 .2500 -8.685E-10  2.895E-10  3.513E-04  3.474E-04  3.969E-05
    98 .2500 -1.775E-10  5.918E-11  1.268E-04  1.250E-04  3.972E-05
    99 .2500 -2.785E-11  9.284E-12  4.319E-05  4.256E-05  3.973E-05
   100 .2500 -5.225E-12  1.742E-12  1.553E-05  1.531E-05  3.973E-05
   101 .2500  0.000E+00  0.000E+00  0.000E+00  0.000E+00  3.973E-05
        -------- THE REST OF THE OUTPUT IS OMITTED -----
```

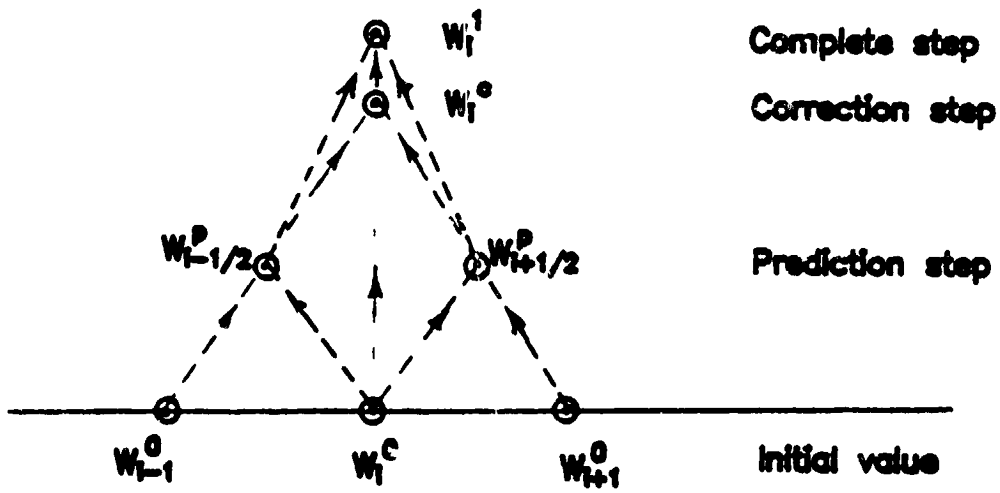
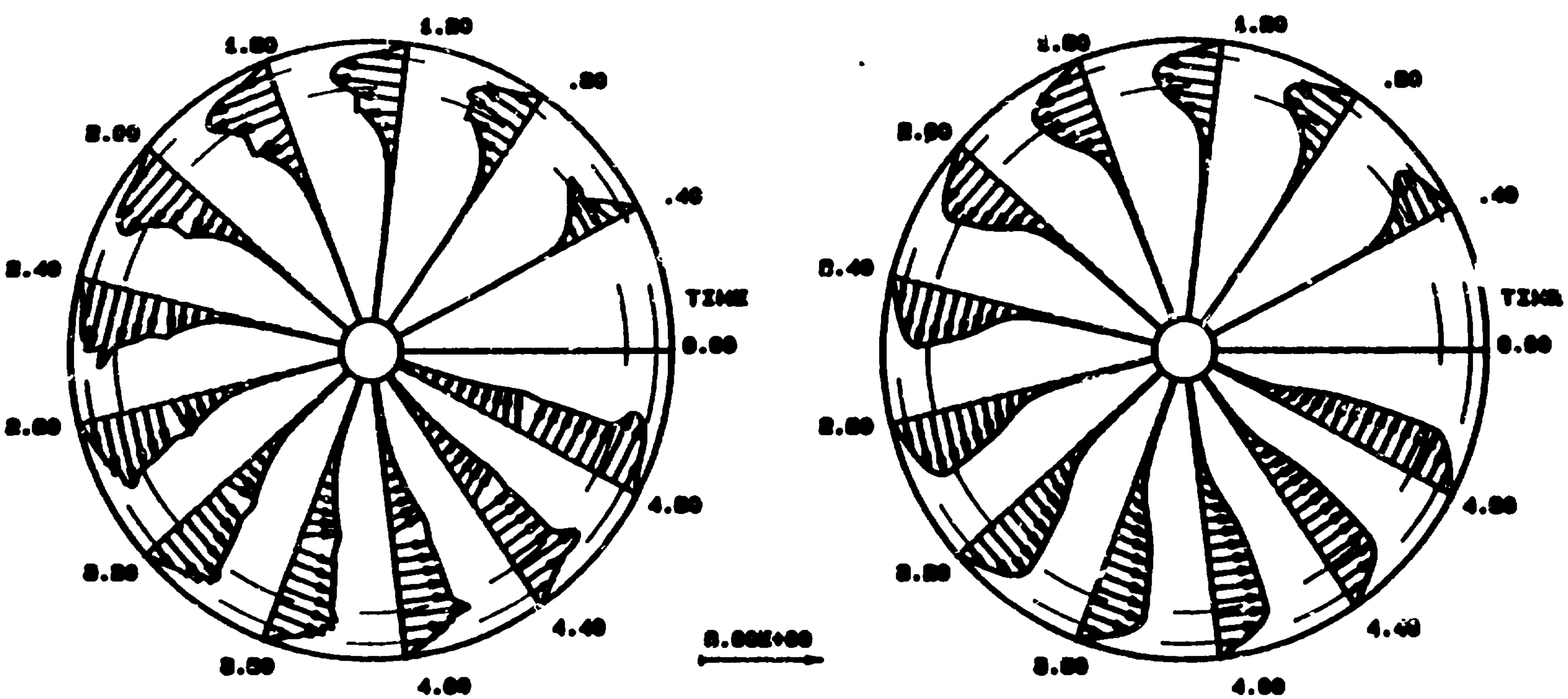


Figure 1. Two Step Difference Scheme (Backward Predictor - Forward Corrector Version)



a. GAS PHASE

b. LIQUID PHASE

Figure 2. Velocity Vector Distributions
The annular region between two dashed circles is the region of injection.

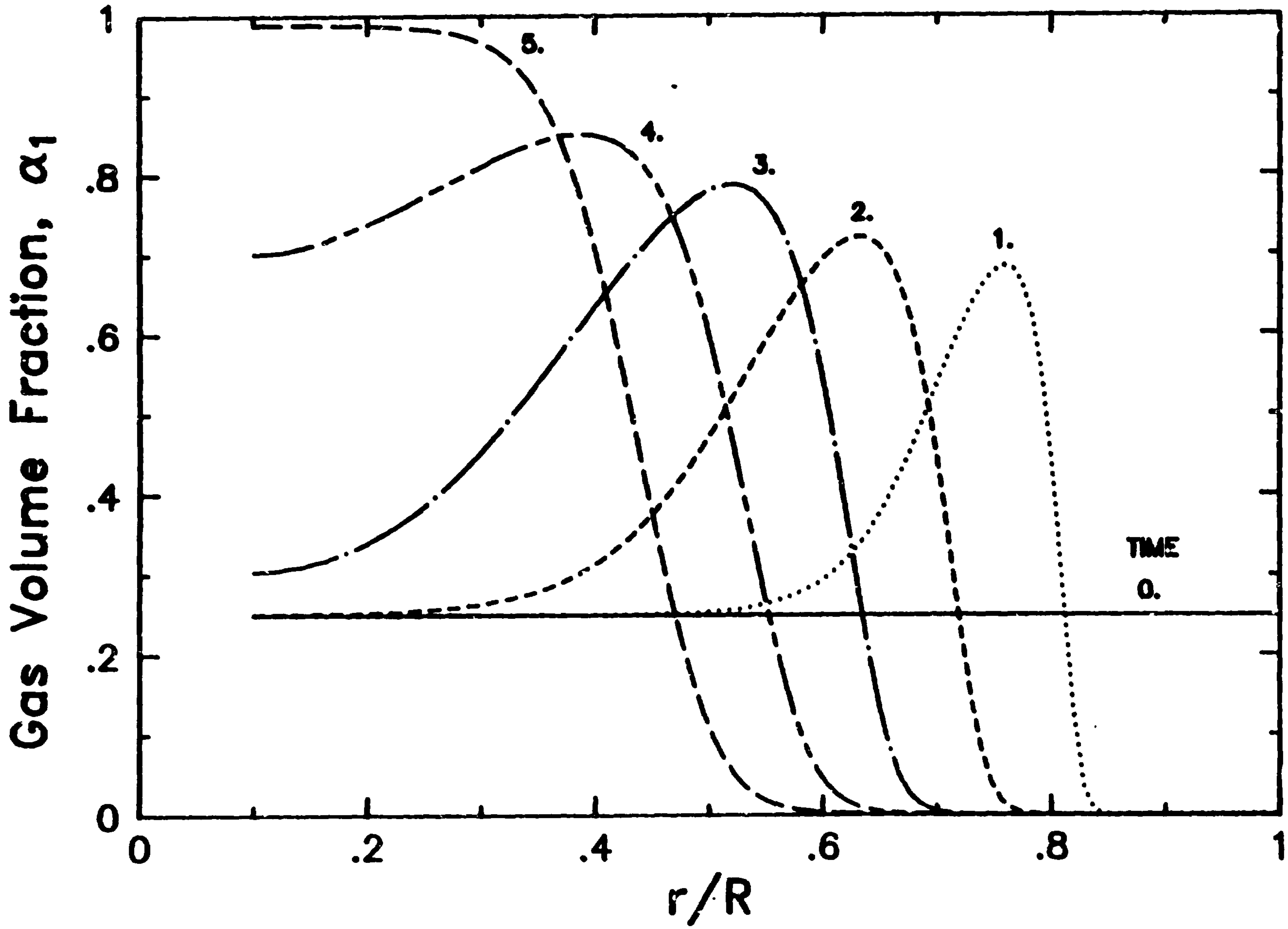


Figure 3. Gas Volume Fraction Distributions